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On the Mayer–Lee–Yang hypothesis for a class of continuous systems

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Abstract. For an arbitrary classical gas of particles in which the interaction is given by a many-body, stable, regular and strongly regular potential we prove the uniqueness and analyticity of the grand canonical tempered Gibbs state for every value of the chemical activity such that $z^{-1} \notin$ {spectrum of the Kirkwood-Salsburg operator}.

1. Introduction

Our knowledge of the structure of the set of Gibbs equilibrium states describing continuous systems of classical particles at thermal equilibrium in the region of large values of the chemical activity or at low temperatures is still very incomplete. Some recent advances include results for a class of generalised Widom-Rowlison type of models (Bricmont *et al* 1984, 1985, Ruelle 1971) and the charged but neutral two-component systems, in which the interaction is given by a sufficiently regular function of positive type (Gielerak 1986, 1987a). This is in contrast to the high-temperature, low-density region where the uniqueness of the tempered Gibbs grand canonical states has been known for a long time (Ruelle 1969).

It was Mayer (1942) who first conjectured the connection between phase transitions and the spectral properties of the Kirkwood-Salsburg operator. Later, Lee and Yang (Yang *et al* 1952) discussed the connection between zeros of the partition function and phase transitions. However, the spectral analysis of the Kirkwood-Salsburg operator, as well as the study of the location of zeros of the partition function, are very difficult and only a few results have been established (Pastur 1974, Moraal 1975, 1977, 1981, Klein 1975, Zagrebnov 1982a, Gorzelanczyk 1985) all of which concern the finite-volume situation or low-activity regions.

In this paper we put the Mayer-Lee-Yang hypothesis on a rigorous mathematical footing. We consider systems of particles, located in R^d at thermal equilibrium, in which the interaction is given by an arbitrary many-body interaction, V, which we assume to be stable and regular in a sense which we will explain below. The corresponding Gibbs distribution is completely described by its correlation functions. Following Moraal (1976) we derive generalised Kirkwood-Salsburg identities between them and we prove in a rigorous mathematical way that for values of the chemical activities, z^{-1}, that do not belong to the spectrum of the corresponding (generalised) Kirkwood-Salsburg operator there exists a unique infinite-volume grand canonical Gibbs state and, moreover, that this state is analytic in z in the sense that all of its moments are analytic. Thus we have localised the set of possible critical values of z as a (subset) of the spectrum of the corresponding Kirkwood–Salsburg operator. This justifies the Mayer hypothesis. The precise formulation of this result is given in § 2 of this paper and the proof is given in § 3.

2. Formulation of the result

Let Ω be the collection of all finite or countable subsets of R^d having no limit points in R^d. Ω is provided with the weakest topology τ in which the map

$$\pi : \Omega \ni \omega \to \omega(\Lambda) \equiv \omega \cap \Lambda \in \Omega_f(\Lambda) \tag{2.1}$$

is continuous for any open bounded Borel subset $\Lambda \subset R^d$, where $\Omega_f(R^d)$ (respectively $\Omega_f(\Lambda)$) is the collection of all finite subsets $\omega \subset R^d$ (respectively $\omega \subset \Lambda$) with the point-to-point convergence topology τ_f. The σ algebra(s) corresponding to τ_f ($\equiv$Borel σ algebra) we will denote by $\mathscr{F}_f(\Omega_f)$ (respectively $\mathscr{F}_f(\Omega_f(\Lambda)) = \mathscr{F}_f(\Lambda)$).

The pair (Ω, τ) then forms a Polish space (Ngyen and Zessin 1979). The corresponding Borel σ algebra is denoted by $\mathscr{F}(R^d)$. In a similar fashion we define the σ algrebras $\mathscr{F}(\Lambda)$, where Λ is an arbitrary Borel subset of R^d. Clearly $\mathscr{F}(\Lambda_1) \subset \mathscr{F}(\Lambda_2)$ provided $\Lambda_1 \subset \Lambda_2$ and moreover, for $\Lambda_1 \cup \Lambda_2$ with $\Lambda_1 \cap \Lambda_2 = \emptyset$, we have $\mathscr{F}(\Lambda_1) \simeq \mathscr{F}(\Lambda_1) \otimes \mathscr{F}(\Lambda_2)$. Let $\hat{\pi}_0^z$ be a corresponding free gas Poisson distribution (not normalised) on the inductive system $(\Omega, \mathscr{F}, \Omega(\Lambda), \mathscr{F}(\Lambda))$ with the chemical activity $z \geqslant 0$. Let V be an arbitrary, real measurable function defined on $\Omega_f(R^d)$ and such that

$$\exists B > 0 \colon \forall \omega \in \Omega_f(R^d) \qquad \mathscr{E}_V(\omega) \equiv \sum_{\substack{\omega' \subset \omega \\ \omega' \neq \emptyset}} V_{\mathrm{card}\,\omega'}(\omega') \geqslant -B \text{ card } \omega \tag{2.2}$$

where $\{V \ldots\}$ is a sequence of real measurable functions which are symmetric on R^{nd} and are associated with V in a standard way, i.e. $V = (V_1, V_2, V_3, \ldots)$.

For a given V as above we also define

$\forall \omega, \omega' \in \Omega_f(R^d)$ and $\omega \cap \omega' = \emptyset$

$$\begin{aligned} \mathscr{E}_V(\omega|\omega') &= \sum_{\substack{\eta \subset \omega \vee \omega' \\ \eta \cap \omega \neq \emptyset \\ \eta \cap \omega' \neq \emptyset}} V_{\mathrm{card}\,\eta}(\eta) \\ &= \mathscr{E}_V(\omega \vee \omega') - \mathscr{E}_V(\omega) - \mathscr{E}_V(\omega'). \end{aligned} \tag{2.3}$$

For $\omega' \in \Omega(R^d)$ we also define

$$\mathscr{E}_V(\omega|\omega') = \lim_{n \to \infty} \mathscr{E}_V(\omega|\omega'(\Lambda_n)) \tag{2.4}$$

where $(\Lambda_n)_n$ is an arbitrary monotonic sequence of bounded subsets of R^d such that $\bigcup_n \Lambda_n = R^d$, if such a limit exists. For $\Lambda \subset R^d$ bounded and $\omega \in \Omega(R^d)$ we define

$$Z_\Lambda^\omega(z, \beta) = \int_{\Omega_f(\Lambda)} \hat{\pi}_0^z(\mathrm{d}\eta) \exp[-\beta \mathscr{E}_V(\eta|\eta \vee \omega(\Lambda^c))] \tag{2.5}$$

$$\mu_\Lambda^\omega(z, \beta|A) = (z_\Lambda^\omega(z, \beta))^{-1} \int_A \hat{\pi}_0^z(\mathrm{d}\eta) \exp[-\beta \mathscr{E}_V(\eta|\eta \vee \omega(\Lambda^c))] \tag{2.6}$$

where

$$\mathscr{E}_V(\eta|\eta \vee \omega(\Lambda^c)) \equiv \mathscr{E}_V(\eta) + \mathscr{E}_V(\eta|\omega(\Lambda^c))$$

for any $A \in \mathscr{F}(\Lambda)$, provided the corresponding limits and integrals under consideration exist.

A probability measure μ on $\{\Omega, \mathscr{F}(R^d)\}$ is called a grand canonical Gibbs (GCG) measure corresponding to the interaction V, chemical activity z ($\geqslant 0$) and (inverse) temperature β iff the following hold.

(*i*) GCG1): the limits (2.4)–(2.6) exist for almost every pair $(\eta, \omega) \in \Omega_f(R^d) \otimes \Omega(R^d)$, with respect to the measure $\pi_0^z \otimes \mu$.

(*ii*) GCG2): in the sense of measures we have

$$\mu \circ \mu_\Lambda^-(z, \beta | \cdot) = \mu$$

for every bounded Borel $\Lambda \subset R^d$, where $(-)$ is the integration variable.

(Of course GCG1) and GCG2) are nothing more than the Dobrushin–Lanford–Ruelle equilibrium equations.)

The set of all such measures we denote by $\mathscr{G}(z, \beta, V)$. It is known that for the case of stable interactions the set $\mathscr{G}(z, \beta, V)$ is non-empty for any $z \geqslant 0$ and $\beta \geqslant 0$ (Preston 1976, Georgii 1979). With additional restrictions on V some uniqueness theorems are known (Dobrushin 1970, Dobrushin and Pecherski 1983, Klein 1982). Finally in the case of superstable interactions it is known that it is possible to select some special subset of $\mathscr{G}(z, \beta, V)$ of the so-called tempered Gibbs measures which has the structure of a Choquet simplex in the weak-convergence topology (Ruelle 1970, Gielerak 1985).

In the following we will restrict the class of admissible interactions V to the case of the so-called R strongly regular interactions which are defined as follows. Let Ψ be an arbitrary montonically decreasing function on $R_+ \equiv \{x \in R | x \geqslant 0\}$ which has the asymptotic form $\Psi(x) \sim x^{-d-\varepsilon}$, for some $\varepsilon > 0$ as $x \uparrow \infty$. The interaction V is called R strongly regular iff for any $\omega_1, \omega_2 \in \Omega_f(R)$ the following estimate holds:

$$|\mathscr{E}_V(\omega_1 | \omega_2)| \leqslant \tfrac{1}{2} \sum_{r,s \in \mathbb{Z}^d} \Psi(|r-s|)(n^2(\omega_1, r) + n^2(\omega_2, s)) \tag{2.7}$$

where $\mathbb{Z}^d$ means the integer lattice and $n(\omega, r)$ denotes the number of particles belonging to the configuration ω that are located in the unit cube $\square_r = \{x \in R^d | r_i \leqslant x_i < r_i + 1\}$.

We will say that the configuration $\omega \in \Omega$ is tempered iff there exists $a > 0$ such that for sufficiently large $|r|$ the following holds: $n(\omega, r) \leqslant a \log |r|$. We denote by $\Omega^T(R^d)$ the set of tempered configurations. It is then clear from (2.7) that for any $|\Lambda| < \infty$ and $\omega' \in \Omega^T(R^d)$ the limit (2.4) exists and is finite for any R strongly regular interactions V. Assuming, moreover, that V is superstable we know that the set $\Omega^T(R^d)$ is of measure one for any infinite-volume superstable solution of GCG1) and GCG2) (Lebowitz and Presutti 1976). For finite volume $|\Lambda| < \infty$ the measure $\mu_\Lambda^\omega(z, \beta | -)$ is completely described by its correlation functions which are defined by

$$\rho_\Lambda^\omega(z, \beta | (x)_n) = (Z_\Lambda^\omega(z, \beta))^{-1} \chi_\Lambda(x)_n \sum_{m=0}^{\infty} \frac{z^{n+m}}{m!} \int_\Lambda \mathrm{d}(y)_m$$
$$\times \exp[-\beta \mathscr{E}_V((x)_n \vee (y)_m | (x_n) \vee (y)_m \vee \omega(\Lambda^c))] \tag{2.8}$$

where we have used standard abbreviations

$$(x)_n = (x_1, \ldots, x_n) \tag{2.9}$$

$$\mathrm{d}(x)_n = \mathrm{d}x_1 \otimes \ldots \otimes \mathrm{d}x_n \tag{2.10}$$

$$\chi_\Lambda(x)_n = \prod_{i=1}^{n} \chi_\Lambda(x_i) \tag{2.11}$$

and χ_Λ is the characteristic function of the set Λ.

It is known that certain sorts of convergence of the correlation functions yield the weak convergence of the corresponding Gibbs measures (Ruelle 1969). In particular, the uniform convergence on compacts (in R^{nd}) induces the weak convergence of the corresponding Gibbs measures.

Following Moraal (1981) we have derived the following identities between the conditioned correlation functions (2.8):

$$\rho_\Lambda^\omega(z,\beta|(x)_n) = z\chi_\Lambda(x)_n \exp[-\beta\mathscr{E}_V^1((x)_n|\omega(\Lambda^c))] \times \sum_{k=1}^{\infty}\frac{1}{k!}\int_\Lambda \mathrm{d}(y)_k \mathscr{H}(x_1|(x)'_n|(y)_k)\rho_\Lambda^\omega(z,\beta|(x)'_n\vee(y)_k) \tag{2.12}$$

for $n>1$, and

$$\rho_\Lambda^\omega(z,\beta|(x_1)) = z\chi_\Lambda(x_1)\exp[-\mathscr{E}_V(x_1|\omega(\Lambda^c))] \times \sum_{k=0}^{\infty}\frac{1}{k!}\int_\Lambda \mathrm{d}(y)_k\mathscr{H}(x_1|\varnothing|(y)_k)\rho_\Lambda^\omega(z,\beta|(y)_k) \tag{2.13}$$

for $n=1$, where we have used

$$(x)'_n = (x_2,\ldots,x_n)$$

$$\mathscr{H}(x_1|(x)'_n|(y)_k) = \sum_{i=0}^{k}\binom{k}{i}(-1)^{k-i}\sigma(x_1|(x)'_n\vee(y)_i) \tag{2.14}$$

$$\sigma(x|(y)_k) = \exp\left(-\beta\sum_{\substack{(\tilde{y})_q\subset(y)_k\\ q\geq 1}}\mathscr{E}_V(x|(\tilde{y})_q)\right) \tag{2.15}$$

where $(\tilde{y})_q$ is an arbitrary q element subset of (y_k) and finally

$$\mathscr{E}_V^1((x)_n|\omega(\Lambda^c)) = \mathscr{E}_V(x_1|(x)'_n\vee\omega(\Lambda^c)). \tag{2.16}$$

According to a well known strategy we will discuss these identities in terms of the resolvent equation in a suitable Banach space which we will now define.

Let $\mathscr{B}_\xi$ be the Banach space consisting of all sequences $\mathbb{F}=(f_n((x)_n))$ where, for each n, f_n is a measurable function equipped with the norm

$$\|\mathbb{F}\|_\xi = \sup_n \xi^{-n} \operatorname*{ess\,sup}_{(x)_n\subset R^{nd}} |f_n((x)_n)| \tag{2.17}$$

where the positive real number $\xi>0$ will be chosen later. In the space $\mathbb{B}_\xi$ let us define the operators $\mathbb{K}_\Lambda^\omega$ and $\mathbb{K}_\infty$ formally by

$$(\mathbb{K}_\Lambda^\omega\mathbb{F})_1((x)_1) = \Pi_\Lambda\exp[-\beta\mathscr{E}_V((x)_1|\omega(\Lambda^c))] \times\sum_{k=1}^{\infty}\frac{1}{k!}\int\mathrm{d}(y)_k\mathscr{H}(x_1|\varnothing|(y)_k)(\Pi_\Lambda\mathbb{F})_k((y)_k) \tag{2.18}$$

$$(\mathbb{K}_\infty\mathbb{F})_1((x)_1) = \sum_{k=1}^{\infty}\frac{1}{k!}\int\mathrm{d}(y)_k\mathscr{H}(x_1|\varnothing|(y)_k)f_k((y)_k) \tag{2.19}$$

where Π_Λ acts in $\mathbb{B}_\xi$ as

$$(\Pi_\Lambda\mathbb{F})_k = \prod_{i=1}^{k}\chi_\Lambda(x_i)f_k((x)_k)$$

and for $n>1$:

$$(\mathbb{K}_\Lambda^\omega \mathbb{F})_n((x)_n)=\Pi_\Lambda \exp[-\beta \mathscr{E}_V^1((x)_n|\omega(\Lambda^c))] \times \sum_{k=1}^{\infty} \frac{1}{k!} \int \mathrm{d}(y)_k \mathscr{H}(x_1|(x)'_n|(y)_k)(\Pi_\Lambda \mathbb{F})_{n+k-1}((x)'_n \vee (y)_k) \tag{2.20}$$

$$(\mathbb{K}_\infty \mathbb{F})_n((x)_n)=\exp[-\beta \mathscr{E}_V^1((x)_n|(x)'_n)] \times \sum_{k=1}^{\infty} \frac{1}{k!} \int \mathrm{d}(y)_k \mathscr{H}(x_1|(x)'_n|(y)_k f_{n+k-1}((x)'_n \vee (y)_k). \tag{2.21}$$

We also define certain vectors from the space $\mathbb{B}_1$:

$$\alpha_\Lambda^\omega(x)_1=\{\exp[-\beta \mathscr{E}_V(x_1|\omega(\Lambda^c))]\chi_\Lambda(x_1), 0, \ldots\} \tag{2.22}$$

$$\alpha_\infty(x)_1=(1,0,0,\ldots). \tag{2.23}$$

Then identities (2.12) and (2.13) can be rewritten formally as

$$\rho_\Lambda^\omega(z,\beta)=z\mathbb{K}_\Lambda^\omega \rho_\Lambda^\omega(z,\beta)+z\alpha_\Lambda^\omega \tag{2.24}$$

where

$$\rho_\Lambda^\omega(z,\beta)=(\rho_\Lambda^\omega(z,\beta|(x_1)), \rho_\Lambda^\omega(z,\beta|(x)_2), \ldots). \tag{2.25}$$

Definition. An interaction V is called M regular iff there exist constants P and Q (possibly depending on β) such that

$$\forall k \geqslant 1, n \geqslant 1: \qquad \sup_{(x)_n} \int |\mathscr{H}(x_1|(x)'_n|(y)_k)| \, \mathrm{d}(y)_k \leqslant Q^k \tag{2.26a}$$

$$\sup_{(x)_n,(y)_k} \exp[-\beta \mathscr{E}_V^1((x)_n|(y)_k)] \leqslant P. \tag{2.26b}$$

Lemma 2.1. Let V be stable and M regular. Then the operator $\mathbb{K}_\infty$ is a bounded operator from $\mathbb{B}_\xi$ to $\mathbb{B}_\xi$ with the following estimate on its norm:

$$\|\mathbb{K}_\infty\|_\xi \leqslant P\xi^{-1} \exp(\xi Q). \tag{2.27}$$

We omit the proof because of its simplicity (see also Moraal 1976).

Now we are ready to formulate our main result. Let $\overline{\mathrm{sp}}(\mathbb{K}_\infty)$ denote the part of the spectrum of the operator $\mathbb{K}_\infty$ (in the space $\mathbb{B}_\xi$) that lies in the set of physical values of z, i.e. on the real semiaxis $\{z \in \mathbb{C}^1 | \mathrm{Im}\, z=0, \mathrm{Re}\, z>0\}=R_+$†.

Theorem. Let V be a stable M regular and R strongly regular many-body potential. Then for every value of the chemical activity z, such that $z \in R_+$ and $z^{-1} \notin \overline{\mathrm{sp}}(\mathbb{K}_\infty)$, there exists at most one tempered grand canonical Gibbs measure in the set $\mathscr{G}^{\mathrm{T}}(z,\beta,V)$. Moreover, this unique state is analytic in z in the sense that all its correlation functions are analytic for $z^{-1} \notin \overline{\mathrm{sp}}(\mathbb{K}_\infty)$.

Corollary 2.2. Let V be additionally superstable. Then for any z: $z^{-1} \notin \mathrm{sp}(\mathbb{K}_\infty)$ the set $\mathscr{G}^{\mathrm{T}}(z,\beta,V)$ consists of exactly one point $\mu_\infty(z,\beta)$ which is analytic at z.

† It is clear that the theorem formulated below is valid for any complex $z^{-1} \notin \mathrm{sp}\mathbb{K}_\infty$. However the physical meaning of the corresponding Gibbs measure for complex or negative values of z is not obvious.

Remark. In the formulation of this theorem we have to use the phrase 'at most one state' because we have such poor control on the support properties of the stable (but not superstable) interactions.

The theorem locates the set of possible values of z which are critical on the spectrum of $\mathbb{K}_\infty$. Note that $\mathbb{K}_\infty$ depends on β so that in general $\overline{\mathrm{sp}}(\mathbb{K}_\infty)$ also depends on β. By means of results established in Moraal (1976, 1977, 1981) and Zagrebnov (1982a) we obtain the following corollaries.

Corollary 2.3. Let $V=(V_1, V_2, 0, 0, \ldots)$ be such that $\exp(-\beta V_1)\in L_1(R^d)$, V_2 is superstable and regular, and such that

$$\mathbb{C}(\beta)=\int |1-\exp(-\beta V_2(x))|\,\mathrm{d}x<\infty. \tag{2.28}$$

Then for any $z\geqslant 0$ there exists a unique tempered Gibbs state in the set $\mathcal{G}^{\mathrm{T}}(z, \beta, V)$ which is analytic in $z\in R_+$.

Remark. In order to prove this corollary we have to modify the definition of the corresponding operators $\mathbb{K}_\Lambda^\omega$, $\mathbb{K}_\infty$ by composing them with the index-juggling operator of Ruelle (1969). So we can say that this is really a corollary of the proof of the theorem rather than an immediate consequence of it.

Corollary 2.4. Let $V=(V_1, V_2, V_3, \ldots)$ be such that $\exp(-V_1)\in L_1(R^d)$, $V_k\geqslant 0$, for $k>1$ and moreover V is R strongly regular. Then for any $z\geqslant 0$ there exists exactly one tempered Gibbs state in the set $\mathcal{G}^{\mathrm{T}}(u, \beta, V)$. Moreover, this state is analytic on the $\mathrm{Re}\, z>0$ axis in the sense that all its correlation functions are analytic there.

3. Proof of the main theorem

In this section we prove our main result stated as the theorem in the previous section, and we outline proofs of the corollaries listed there. Let us start with an exposition of the main ideas involved in the proof presented below. As a starting point we use the observation that the operators $\mathbb{K}_\Lambda^\omega$ and $\mathbb{K}_\Lambda$ differ only multiplicatively by a factor which depends only on the energy of the configuration of particles located at Λ with the configuration $\omega(\Lambda^c)$. From the R strong regularity of the interaction V it then follows that locally this factor tends to zero in a suitable sense (see lemma 3.1 below). Then we look at the duals of the operators $\mathbb{K}_\Lambda^\omega$ and $\mathbb{K}_\Lambda$ (see (3.10)) and we find that in the limit $\Lambda=R^d$ the corresponding dual operators $^*\mathbb{K}_\infty^\omega$ and $^*\mathbb{K}_\infty$ exist, $^*\mathbb{K}_\infty^\omega={}^*\mathbb{K}_\infty$ and moreover the convergence $^*\mathbb{K}_\Lambda^\omega\to{}^*\mathbb{K}_\infty$ is in the sense of strong operator convergence. This yields the strong convergence of the corresponding resolvents. In this way we obtain the convergence $\rho_\Lambda^\omega\to\rho_\infty$ in the weak-* topology of the space $\mathbb{B}_\xi$. Then this convergence is increased to the componentwise, uniform convergence on compacts and this yields the weak convergence of the corresponding Gibbs measures. The last step is achieved by using the Mayer-Montroll equations.

Remark. The idea of using dual space techniques appeared first in Ruelle (1970) and then was applied in a more explicit form in Zagrebnov (1982b). For other applications to similar problems see also Gielerak (1986, 1987a, 1987b).

The Banach space $\mathbb{B}_\xi$ introduced above is the dual of the Banach space $^*\mathbb{B}_\xi$ consisting of all sequences of measurable functions $\Psi=(\psi_n(x)_n)$ equipped with the norm

$$^*\|\Psi\|_\xi=\sum_{n=1}^{\infty}\xi^n\int \mathrm{d}(y)_n|\psi_n(x)_n|. \tag{3.1}$$

Then the pair of Banach spaces $(^*\mathbb{B}_\xi, \mathbb{B}_\xi)$ forms a dual pair of Banach spaces.

In the space $\mathbb{B}_\xi$ let us define a linear bounded operator k_∞ by the following formula:

$$(k_\infty \mathbb{F})_n(x)_n=\sum_{l=1}^{\infty}\frac{1}{l!}\int \mathrm{d}(y)_l \mathscr{K}(x_1|(x)'_n|(y)_l)f_{n+l-1}((x)'_n\vee(y)_l). \tag{3.2}$$

Then we have

$$\begin{aligned}\mathbb{K}^\omega_\Lambda&=\Pi_\Lambda\circ\exp[-\beta\mathscr{E}^1_V(\cdot|\omega(\Lambda^c))]\circ k_\infty\circ\Pi_\Lambda\\ &\equiv\Pi_\Lambda\circ\exp[-\beta\mathscr{E}^1_V(\cdot|\omega(\Lambda^c))]\circ k_\Lambda\end{aligned} \tag{3.3}$$

and also

$$\mathbb{K}_\infty=\exp[-\beta\mathscr{E}^1_V((x)_n)]\circ k_\infty \tag{3.4}$$

where $\exp[-\beta\mathscr{E}^1_V((x)_n|\omega(\Lambda^c))]$ acts as a multiplication operator and Π_Λ is defined by

$$(\Pi_\Lambda\mathbb{F})_n((x)_n)=\prod_{i=1}^{n}\chi_\Lambda(x_i)f_n((x)_n). \tag{3.5}$$

We show that there exist operators $^*k_\Lambda$ and $^*k_\infty$ acting in the space $^*\mathbb{B}_\xi$ such that $\forall\psi\in{}^*\mathbb{B}_\xi$ and $\mathbb{F}\in\mathbb{B}_\xi$:

$$\begin{aligned}\langle{}^*k_\Lambda\psi,\mathbb{F}\rangle&=\langle\psi,k_\Lambda\mathbb{F}\rangle\\ \langle{}^*k_\infty\psi,\mathbb{F}\rangle&=\langle\psi,k_\infty\mathbb{F}\rangle\end{aligned} \tag{3.6}$$

and also

$$\langle\{{}^*k_\Lambda\circ\exp[-\beta\mathscr{E}^1_V(\ldots|\omega(\Lambda^c))]\circ\Pi_\Lambda\}\psi,\mathbb{F}\rangle=\langle\psi,\mathbb{K}^\omega_\Lambda\mathbb{F}\rangle. \tag{3.7}$$

Similarly

$$\langle{}^*k_\infty\circ\exp[-\beta\mathscr{E}^1_V(\ldots)]\psi,\mathbb{F}\rangle=\langle\Psi,\mathbb{K}_\infty\mathbb{F}\rangle \tag{3.8}$$

where $\langle\Psi,\mathbb{F}\rangle$ means the canonical pairing, i.e.

$$\langle\Psi,\mathbb{F}\rangle=\sum_{n=1}^{\infty}\int\mathrm{d}(x)_n\Psi_n((x)_n)f_n((x)_n). \tag{3.9}$$

A simple calculation quickly verifies that the following expression for the dual operator $^*k_\infty$ is valid:

$$({}^*k_\infty\Psi)((x)_n)=\sum_{l=0}^{n}\frac{1}{l!}\int\mathrm{d}(y)\mathscr{K}(y|(x)_m-(x)_l|(x)_l)\Psi_{n+1-l}(y\vee((x)_m-(x)_l)). \tag{3.10}$$

Lemma 3.1. Assume that V is R strongly regular. Let $\omega\in\Omega^{\mathrm{T}}(R^d)$ and let us denote

$$\boldsymbol{\mathscr{E}}^1(\omega(\Lambda^c))=\left[\left(\prod_{i=1}^{n}\chi_\Lambda\right)(x)_n\exp[-\beta\mathscr{E}^1((x_1)|(x)'_n\omega(\Lambda^c))]\right]_{n=1,2,\ldots} \tag{3.11}$$

and

$$\boldsymbol{\mathscr{E}}^1=[\exp[-\beta\mathscr{E}^1(\omega=\varnothing)]]_{n=1,2,\ldots}. \tag{3.12}$$

Then for any compact $\Delta\subset R^d$ we have

$$\lim_{\Lambda\uparrow R^d}\|\Pi_\Delta(\boldsymbol{\mathscr{E}}^1(\omega(\Lambda^c))-\boldsymbol{\mathscr{E}}^1)\|_\xi=0. \tag{3.13}$$

Proof. Taking $\omega \in \Omega^{\mathrm{T}}(R^d)$ we note that

$$\begin{aligned}|(\mathscr{E}^1(\omega(\Lambda)))_n| &= |\mathscr{E}_V((x_1|(x)'_n \vee \omega(\Lambda^c))| \\ &\leqslant \tfrac{1}{2} \sum_{\Gamma' \in \mathbb{Z}^d} \Psi(|r|) + \tfrac{1}{2} a' \sum_{r \in \Lambda, s \in \Lambda^c} \Psi(|r-s|) \log^2|s|\end{aligned} \tag{3.14}$$

for some constant a' and for fixed r, from which it follows that

$$\sup_{|\Lambda|} |(\mathscr{E}^1(\omega(\Lambda)))_n| < \infty \tag{3.15}$$

and now the claim follows easily from the assumed decay of Ψ.

Lemma 3.2. Let V be regular in the sense of (2.26) and R strongly regular. Then for any $\omega \in \Omega^{\mathrm{T}}(R^d)$ we have

$$s - \lim_{\Lambda \uparrow R^d} ({}^*\mathbb{K}^{\omega}_{\Lambda} - {}^*\mathbb{K}_{\infty}) = 0. \tag{3.16}$$

Proof. For any $\psi \in {}^*\mathbb{B}_{\xi}$ we have

$$\begin{aligned}{}^*\|({}^*\mathbb{K}^{\omega}_{\Lambda} - {}^*\mathbb{K}_{\infty})\psi\|_{\xi} &\leqslant {}^*\|{}^*k_{\infty}\{\exp[-\beta\mathscr{E}^1(-|\omega(\Lambda^c))]\Pi_{\Lambda} - \exp[-\beta\mathscr{E}^1(-)]\}\psi\|_{\xi} \\ &\leqslant \|\|{}^*k_{\infty}\|\|_{\xi} \cdot {}^*\|\{\exp[-\beta\mathscr{E}^1(-|\omega(\Lambda^c))]\Pi_{\Lambda} - \exp[-\beta\mathscr{E}^1(-)]\}\psi\|_{\xi}.\end{aligned}$$

For any $\psi \in {}^*\mathbb{B}_{\xi}$ and any $\varepsilon > 0$ there exists a bounded measurable subset $\Delta_{\varepsilon} \subset R^d$, for which ${}^*\|\psi - \Pi_{\Delta_{\varepsilon}}\psi\|_{\xi} < \varepsilon$. Therefore we have

$$\begin{aligned}&{}^*\|\{\exp[-\beta\mathscr{E}^1((x)_n|\omega(\Lambda^c))]\Pi_{\Lambda} - \exp[-\beta\mathscr{E}^1((x)_n)]\}\psi\|_{\xi} \\ &\quad\leqslant {}^*\|\{\exp[-\beta\mathscr{E}^1((x)_n|\omega(\Lambda^c))]\Pi_{\Lambda} - \exp[-\beta\mathscr{E}^1((x)_n))]\Pi_{\Lambda_{\varepsilon}}\}\psi\|_{\xi} \\ &\qquad + {}^*\|\{\exp[-\beta\mathscr{E}^1((x)_n|\omega(\Lambda^c))]\Pi_{\Lambda} - \exp[-\beta\mathscr{E}^1((x)_n)](1-\Pi_{\Lambda_{\varepsilon}})\}\psi\|_{\xi} \\ &\quad\leqslant \|(\mathscr{E}^1(\omega(\Lambda^c)) - \mathscr{E}^1)\Pi_{\Lambda_{\varepsilon}}\|_{\xi} \cdot {}^*\|\psi\|_{\xi} + \|\mathscr{E}^1(\omega(\Lambda^c)) - \mathscr{E}^1\|_{\xi} \cdot {}^*\|(1-\Pi_{\Lambda_{\varepsilon}})\psi\|_{\xi}.\end{aligned}$$

From lemma 3.1, (3.15) and our choice of the set Δ_{ε} it follows that in the limit $\Lambda = R^d$ we get

$$\lim_{\Lambda \uparrow R^d} {}^*\|{}^*\mathbb{K}^{\omega}_{\Lambda} - \mathbb{K}_{\infty}\psi\|_{\xi} = 0.$$

The strong convergence proved in lemma 3.2 sometimes yields the strong convergence of the corresponding resolvents in the space ${}^*\mathbb{B}_{\xi}$.

Corollary 3.3. For any $z \in \mathbb{C}$: $z^{-1} \notin \mathrm{sp}({}^*\mathbb{K}_{\infty})$ and any $\omega \in \Omega^{\mathrm{T}}(R^d)$ we have the strong convergence

$$s - \lim_{\Lambda \uparrow R^d} (1 - z{}^*\mathbb{K}^{\omega}_{\Lambda})^{-1} = (1 - z{}^*\mathbb{K}_{\infty})^{-1} \tag{3.17}$$

assuming the potential V is regular as before and that the spectrum of $\mathbb{K}^{\omega}_{\Lambda}$ does not contract†. From the Phillips theorems we know that

$$\mathrm{sp}\,{}^*\mathbb{K}^{\omega}_{\Lambda} = \mathrm{sp}\,\mathbb{K}^{\omega}_{\Lambda} \qquad \mathrm{sp}\,{}^*\mathbb{K}_{\infty} = \mathrm{sp}\,\mathbb{K}_{\infty} \tag{3.18}$$

† Taking into account the following formula:

$$(1 - z{}^*\mathbb{K}^{\omega}_{\Lambda})^{-1} = (1 - z{}^*\mathbb{K}_{\infty})^{-1}[1 + z({}^*\mathbb{K}_{\infty} - {}^*\mathbb{K}^{\omega}_{\Lambda})(1 - z{}^*\mathbb{K}_{\infty})^{-1}]^{-1}$$

it follows that for Λ sufficiently large to make $z\|{}^*\mathbb{K}^{\omega}_{\Lambda} - {}^*\mathbb{K}_{\infty}\|$ less than the norm of $(1 - z{}^*\mathbb{K}_{\infty})^{-1}$ which, by definition, is a bounded operator whenever $z^{-1} \notin \mathrm{sp}\,\mathbb{K}_{\infty}$, we see that the hypothesis about the spectrum of $\mathbb{K}^{\omega}_{\Lambda}$ is not necessary whenever ${}^*\mathbb{K}^{\omega}_{\Lambda} \to {}^*\mathbb{K}_{\infty}$ in the norm. This was pointed out by a referee and is well known.

and for $z^{-1} \notin \operatorname{sp} \mathbb{K}_\Lambda^\omega$ we have

$$[(1-z\mathbb{K}_\Lambda^\omega)^{-1}]^* = (1-z^*\mathbb{K}_\Lambda^\omega)^{-1} \tag{3.19}$$

and for $z^{-1} \notin \operatorname{sp} \mathbb{K}_\infty$ we have

$$[(1-z\mathbb{K}_\infty)^{-1}]^* = (1-z^*\mathbb{K}_\infty)^{-1}. \tag{3.20}$$

For the Phillips theorems see Yosida (1966). We are now ready to prove the following proposition.

Proposition 3.4. Let V be regular in the sense of (2.26) and R strongly regular, and let z be such that $z^{-1} \notin \operatorname{sp} \mathbb{K}_\infty$. Then for any $\omega \in \Omega^{\mathrm{T}}(R^d)$ we have

$$w\text{-}*\lim_{\Lambda \uparrow R^d} \rho_\Lambda^\omega(z,\beta) = w\text{-}*\lim_{\Lambda \uparrow R^d} \rho_\Lambda^{\omega=\varnothing}(z,\beta) \tag{3.21}$$

$$\equiv \rho_\infty(z,\beta) \stackrel{df}{\equiv} z(1-z\mathbb{K}_\infty)^{-1}\alpha_\infty.$$

where w-$*$ means convergence in weak-$*$ topology of the space $\mathbb{B}_\xi$.

Proof. Firstly, suppose that the spectrum of $\mathbb{K}_\Lambda^\omega$ does not contract. Taking $\psi \in {}^*\mathbb{B}_\xi$ in an arbitrary way we have

$$\begin{aligned}
&\lim_{\Lambda \uparrow R^d} \langle \psi, \rho_\Lambda^\omega(z,\beta) - \rho_\infty(z,\beta)\rangle \\
&\quad = \lim_{\Lambda \uparrow R^d} [\langle \psi, (1-z\mathbb{K}_\Lambda^\omega)^{-1}\alpha_\Lambda^\omega - (1-z\mathbb{K}_\infty)^{-1}\alpha_\infty\rangle] \\
&\quad = \lim_{\Lambda \uparrow R^d} \{\langle \psi, [(1-z\mathbb{K}_\Lambda^\omega)^{-1} - (1-z\mathbb{K}_\infty)^{-1}]\alpha_\infty\rangle + \langle \psi, (1-z\mathbb{K}_\Lambda^\omega)^{-1}(\alpha_\Lambda^\omega - \alpha_\infty)\rangle\} \\
&\quad = \lim_{\Lambda \uparrow R^d} \{\langle [(1-z^*\mathbb{K}_\Lambda^\omega)^{-1} - (1-{}^*z\mathbb{K}_\infty)^{-1}]\psi, \alpha_\infty\rangle + \langle (1-z^*\mathbb{K}_\Lambda^\omega)^{-1}\psi, \alpha_\Lambda^\omega - \alpha_\infty\rangle\} \\
&\quad = 0
\end{aligned}$$

where we have applied corollary 3.3 and the simple verifiable fact that $\alpha_\Lambda^\omega \to \alpha_\infty$ in the weak-$*$ topology of the space $\mathbb{B}_\xi$. However, in general the spectrum of the operator K_Λ^ω can contract (Reed and Simon 1972). In this case we proceed in the following way. From the Kirkwood-Salsburg identities (2.24), (3.21) and the definitions of the dual operators we get

$$\bigvee_{\Psi \in {}^*\mathbb{B}_\xi} \quad \langle (1-z^*\mathbb{K}_\infty)\Psi, (\rho_\Lambda^\omega - \rho_\infty)\rangle = \langle z({}^*\mathbb{K}_\Lambda^\omega - {}^*\mathbb{K}_\infty)\psi, \rho_\Lambda^\omega\rangle + z\langle \Psi, \alpha_\Lambda^\omega - \alpha_\infty\rangle. \tag{3.22}$$

From the regularity of the interaction and the Banach-Alaoglou theorem it follows easily that $\{\rho_\Lambda^\omega\}$ forms a $*$-weakly precompact set in $\mathbb{B}_\xi$. Let ρ'_∞ be any of the accumulation points of this set. Assuming $z^{-1} \notin \operatorname{sp} K$, then from lemma 3.2, (3.22) and the Phillips theorem, and by the fact that $\alpha_\Lambda^\omega \to \alpha_\infty$ in the weak-$*$ topology, it follows that $\rho_\Lambda^\omega \to \rho_\infty$ and, moreover, that the convergence has to be understood in the weak-$*$ topology of the space $\mathbb{B}_\xi$.

Repeating the procedure which led us to the Kirkwood-Salsburg identities we derive the Mayer-Montroll-like identities, for $\rho_\Lambda^\omega(z,\beta)$:

$$\begin{aligned}
\rho_\Lambda^\omega(z,\beta|(x)_n) &= z\chi_\Lambda(x)_n \exp[-\beta\mathscr{E}_V^{(n)}((x)_n|\omega(\Lambda^c))] \\
&\quad \times \sum_{k=0}^\infty \frac{1}{k!}\int_\Lambda \mathrm{d}(y)_k \mathscr{M}((x)_n|(y)_k)\rho_\Lambda^\omega(z,\beta|(y)_k)
\end{aligned} \tag{3.23}$$

and for $\rho_\infty(z,\beta)$:

$$\rho_\infty(z,\beta|(x)_n)=z\exp[-\beta\mathscr{E}_V((x)_n)]\sum_{k=0}^{\infty}\frac{1}{k!}\int d(y)_k\mathscr{M}((x)_n|(y)_k)\rho_\Lambda^\omega(z,\beta|(y)_k) \qquad (3.24)$$

where the Mayer-Montroll kernels $\mathscr{M}(\cdot|\cdot)$ are given by the following formulae:

$$\mathscr{M}((x)_n|(y)_k)\equiv\sum_{q=0}^{k}\binom{k}{q}(-1)^{k-q}\exp\Bigg(\sum_{\substack{(x)_k\subset(x)_n\\(y)_l\subset(y)_k\\k\geq1,l\geq1}}-\beta V_{k+l}((x)_k\vee(y)_l)\Bigg). \qquad (3.25)$$

Lemma 3.5. For each fixed $(x)_n\in R^{dn}$ the map

$$(x_n)\to[(1/k!)\mathscr{M}((x)_n|(y)_k)]_{k=1,2,\ldots}$$

takes values in the space $^*\mathbb{B}_\xi$ and we have the following estimate on its norm:

$$^*\|[(1/k!)\mathscr{M}((x)_n|(y)_k)]_{k=1,2,\ldots}\|_\xi\leq P^n\exp(O(1)Q). \qquad (3.26)$$

Proof. We start with the estimate

$$\int_{R^d}|\mathscr{M}((x)_n|(y)_k)|\,d(y)_k\leq O(1)P^nQ^k \qquad (3.27)$$

where c is some constant. From this the estimate (3.26) easily follows using the definition of the norm $*\|\cdot\|_\xi$. To prove (3.27) we observe that the Mayer-Montroll kernel can be rewritten in the following way:

$$M(X_n|X_m)=\sum_{q=0}^{M}\binom{m}{q}(-1)^{m-q}\prod_{i=1}^{N}\exp\Bigg(-\beta\sum_{\substack{X_p\subset X_n\\Y_r\subset Y_q\\p\geq1,r\geq1}}^{\leq i}V_{p+r}(X_p\vee Y_r)\Bigg) \qquad (3.28)$$

where we have denoted

$$\sum_{\substack{X_p\subset X_r\\Y_r\subset Y_q\\p\geq1,r\geq1}}^{\leq i}\ldots=\sum_{\substack{X_p\subset X_n\\Y_r\subset Y_q\\x_1,\ldots,x_{i-1}\notin X_p\\x_i\in X_p\\p\geq1,r\geq1}}\ldots. \qquad (3.29)$$

Then we have

$$M(X_n|Y_m)=K(X_n,Y_m) \qquad (3.30)$$

$$=\sum_{q=0}^{m}\binom{m}{q}(-1)^{m-q}\exp\Bigg(-\beta\sum_{\substack{X_p\subset X_n\\Y_r\subset Y_q\\p\geq1,r\geq1}}^{\leq1}V_{p+r}(X_p\vee Y_r)\Bigg)$$

$$\times\sum_{i=2}^{N}\prod_{k=2}^{i-1}\exp\Bigg(-\beta\sum_{\substack{X_p\subset X_n\\Y_r\subset Y_q\\p\geq1,r\geq1}}^{\leq i-1}V_{p+r}(X_p\vee Y_r)\Bigg)$$

$$\times\Bigg[\exp\Bigg(-\beta\sum_{\substack{X_p\subset X_n\\Y_r\subset Y_q\\p\geq1,r\geq1}}^{\leq i}V_{p+r}(X_p\vee Y_r)\Bigg)-1\Bigg]. \qquad (3.31)$$

Now we use the observation that the sum $\Sigma^{\leq i}$ gives exactly the interaction energy of the ith particle with the group of particles composed of $(X_n-(x_1,\ldots,x_{i-1}))\vee Y_q$. Therefore using assumption (2.26*b*) on the potential V we can bound

$$\left|\sum_{i=2}^{n}\prod_{k=2}^{i=1}\exp\left(-\beta\sum_{\substack{X_p\subset X_n\\ Y_r\subset Y_q\\ p\geq 1,r\geq 1}}^{\leq i-1}V_{p+r}(X_p\vee Y_r)\right)\left[\exp\left(-\beta\sum_{\substack{X_p\subset X_n\\ Y_r\subset Y_q\\ p\geq 1,r\geq 1}}^{\leq i}V_{p+r}(X_p\vee Y_r)\right)-1\right]\right|$$

$$\leq\sum_{i=2}^{n}(P^i+P^{i-1})\leq O(1)P^n. \tag{3.32}$$

Using assumption (2.26*a*) we can estimate the difference:

$$\left|\int d(y)_m(\mathcal{M}(X_n|Y_m)-K(X_n|Y_m))\right|\leq O(1)P^nQ^m \tag{3.33}$$

from which the estimate (3.27) then follows.

Now we are ready to prove the basic convergence result.

Proposition 3.6. Let V be regular in the sense of (2.26) and R strongly regular, and let $\omega\in\Omega^{T}(R^d)$. Let $z\in C^1$ be such that $z^{-1}\notin \operatorname{sp}\mathbb{K}_\infty$. Then for any compact $\Delta\subset R^{md}$ and any $n\geq 1$ we have

$$\lim_{\Lambda\uparrow R^d}\|\chi_\Delta(x)_n(\rho^{\omega}_{\Lambda}(z,\beta|(x)_n)-\rho_\infty(z,\beta|(x)_n))\|_{L^\infty(R^{md})}=0. \tag{3.34}$$

Proof. Calculate the difference $\rho^{\omega}_{\Lambda}(z,\beta|(x)_n)-\rho_\infty(z,\beta|(x)_n)$ in terms of the Mayer-Montroll identities (3.23) and (3.24) and then use lemma 3.4 together with lemma 3.5 and the easily verifiable fact that

$$\lim_{\Lambda\uparrow R^d}\|\chi_\Delta\{\exp[-\beta\mathscr{E}((x)_n|(x)_n\vee\omega(\Lambda^c))]-\exp[-\beta\mathscr{E}((x)_n|(x)_n)]\}\|_{L^\infty(R^{md})}=0 \tag{3.35}$$

for any $\omega\in\Omega^{T}(R^d)$ and the potential V fulfilling our regularity properties.

Remark. In the case when each V_k is also continuous in its variables $(x)_k$ we can replace the L^∞ norms by the sup and this shows that in this case we have standard uniform convergence on compacts. Taking into account the introductory remarks at the beginning of this section, we see that we have essentially finished the proof of the theorem.

Let us proceed to prove corollary 2.3 and corollary 2.4 from § 2.

Proof of corollary 2.3 and corollary 2.4. The following lemmas proven in Moraal (1975, 1977, 1981) combined with the theorem gives the proofs.

Lemma 3.7. Let $V=(V_1,V_2,0,\ldots)$ be stable, regular and R strongly regular, and such that $\exp(-\beta V_1)\in L_1(R^d)$. Then the spectrum of the corresponding Kirkwood-Salsburg operator is given by

$$\operatorname{sp}\mathbb{K}_\infty=\{z\in\mathbb{C}^1|Z_\infty(z^{-1},\beta)=0\}. \tag{3.36}$$

Proof. See Moraal (1975).

Lemma 3.8 (Moraal 1981). Let $V=(V_1, V_2, \ldots, V_k, \ldots)$ be stable, $\exp(-\beta V_1)\in L_1(R^d)$, $V_k \geqslant 0$ for $k>1$, and moreover let V be R strongly regular. Then the spectrum of the corresponding Kirkwood-Salsburg operator coincides with the set $\{z\in \mathbb{C}^1 | Z_\infty(z^{-1}, \beta)=0\}$.

A simple application of the Jensen inequalities gives $Z_\infty(z^{-1}, \beta)>0$ for any $z\in R_+$, and from the assumption $\exp(-\beta V_1)\in L_1(R^d)$ and the stability of V it follows that $Z_\infty(z, \beta)<\infty$ for any $z\in\mathbb{C}^1$ in both cases.

For the proof of corollary 2.3 we have to redefine the Kirkwood-Salsburg operator in order for V to fulfil the regularity condition (2.26). According to Ruelle there exists an index-juggling operator $\mathcal{M}$ choosing the index of the first particle x_1 in such a way that we have

$$\sum_{i=2}^{n} V_2(x_1-x_i)\geqslant -2B \tag{3.37}$$

where B is the stability constant of V_2. Then for the operator $\tilde{\mathbb{K}}_\infty=\mathbb{K}_\infty\cdot\mathcal{M}$ the regularity condition (2.26) is fulfilled. For (2.26a) we have to assume that

$$C(\beta)=\int \mathrm{d}y|\exp(-\beta V(y))-1|<\infty \tag{3.38}$$

but this follows from the assumed strong R-regularity of V. The spectrum of the operator $\tilde{\mathbb{K}}_\Lambda$ with $V_1=0$ but $|\Lambda|<\infty$ has been investigated by Zagrebnov (1982) and there it is proved that the union of the point spectrum and generalised eigenvalues of the operator $\tilde{\mathbb{K}}_\Lambda$ is contained in the set $\{z\in\mathbb{C}^1|Z_\Lambda(z^{-1}, \beta)=0\}$, and for the case of a superstable interaction V_2 the remaining piece of the spectrum, i.e. the residual spectrum, disappears. His analysis carries over into our case with the assumption $\exp(-\beta V)\in L^1(R^d)$. Finally, the proof of the existence of the dual operators $(\tilde{\mathbb{K}}_\Lambda^\omega)^*$, $(\tilde{\mathbb{K}}_\infty)^*$ and its strong convergence has been demonstrated in Zagrebnov (1982) and Gielerak (1987b).

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